

Name: _____

Grade: XII

Holiday-Work



Subject: MATHEMATICS

Each Carries One marks

- Q. 1 Let $A = \{1, 2, 3\}$ and let $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)\}$. Then, R is
- Q. 2 Let S be the set of all straight lines in a plane. Let R be a relation on S defined by $a R b \iff a \parallel b$. Then, R is
- Q. 3 Let S be the set of all real numbers and let R be a relation on S , defined by $a R b \iff (1 + ab) > 0$. Then, R is
- Q. 4 Show that $\cos^{-1}\left\{\frac{-\pi}{3}\right\} \neq \frac{-\pi}{3}$ and find its value.
- Q. 5 Show that $\sin^{-1}\left\{\frac{3\pi}{4}\right\} \neq \frac{3\pi}{4}$ and find its value.
- Q. 6 Show that $\tan^{-1}\left\{\frac{5\pi}{6}\right\} \neq \frac{5\pi}{6}$ and find its value
- Q. 7 Evaluate: $\sin\left\{\frac{\pi}{3} - \sin^{-1}\frac{-1}{2}\right\}$
- Q. 8 If a matrix has 6 elements, write all possible order it can have?
- Q. 9 If order of matrix A is 2×3 and order of matrix B is 3×4 , find the order of AB .
- Q. 10 Write the value of $x + y + z$ if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Each Carries Two marks

- Q. 11 Let $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a + 3b = 12\}$. Find the domain and range of R .
- Q. 12 Let $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } b = a + 5, a < 4\}$. Find the domain and range of R .
- Q. 13 Let R_0 be the set of all nonzero real numbers. Show that $f : R_0 \rightarrow R_0 : f(x) = \frac{1}{x}$ is a one-one onto function.
- Q. 14 Show that the function $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = 3 - 4x$ is one-one onto and hence bijective.
- Q. 15 Let A and B be two nonempty sets. Show that the function $f : (A \times B) \rightarrow (B \times A) : f(a, b) = (b, a)$ is a bijective function.
- Q. 16
If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, find $a + b + c$.
- Q. 17 Evaluate: $\cos\left\{\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right\}$
- Q. 18 Evaluate: $\tan^{-1}\left\{2 \cos\left(2 \sin^{-1}\frac{1}{2}\right)\right\}$
- Q. 19 Prove that: $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

Q. 20 Prove that: $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Each Carries three marks

Q. 21 Let A be the set of all points in a plane and let O be the origin. Show that the relation $R = \{(P, Q) : P, Q \in A \text{ and } OP = OQ\}$ is an equivalence relation.

Q. 22 Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$, defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

is one-one and onto.

Q. 23 Show that the signum function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is neither one-one nor onto.

Q. 24 Prove that: $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

Q. 25 Prove that: $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Q. 26 Prove that: $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

Q. 27 Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

Q. 28 Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Q. 29

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that $F(x) \cdot F(y) = F(x + y)$

Q. 30

Using elementary operation, find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$